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DYNAMIC WALL PRESSURE MEASUREMENTS

by

PATRICK LEEHEY

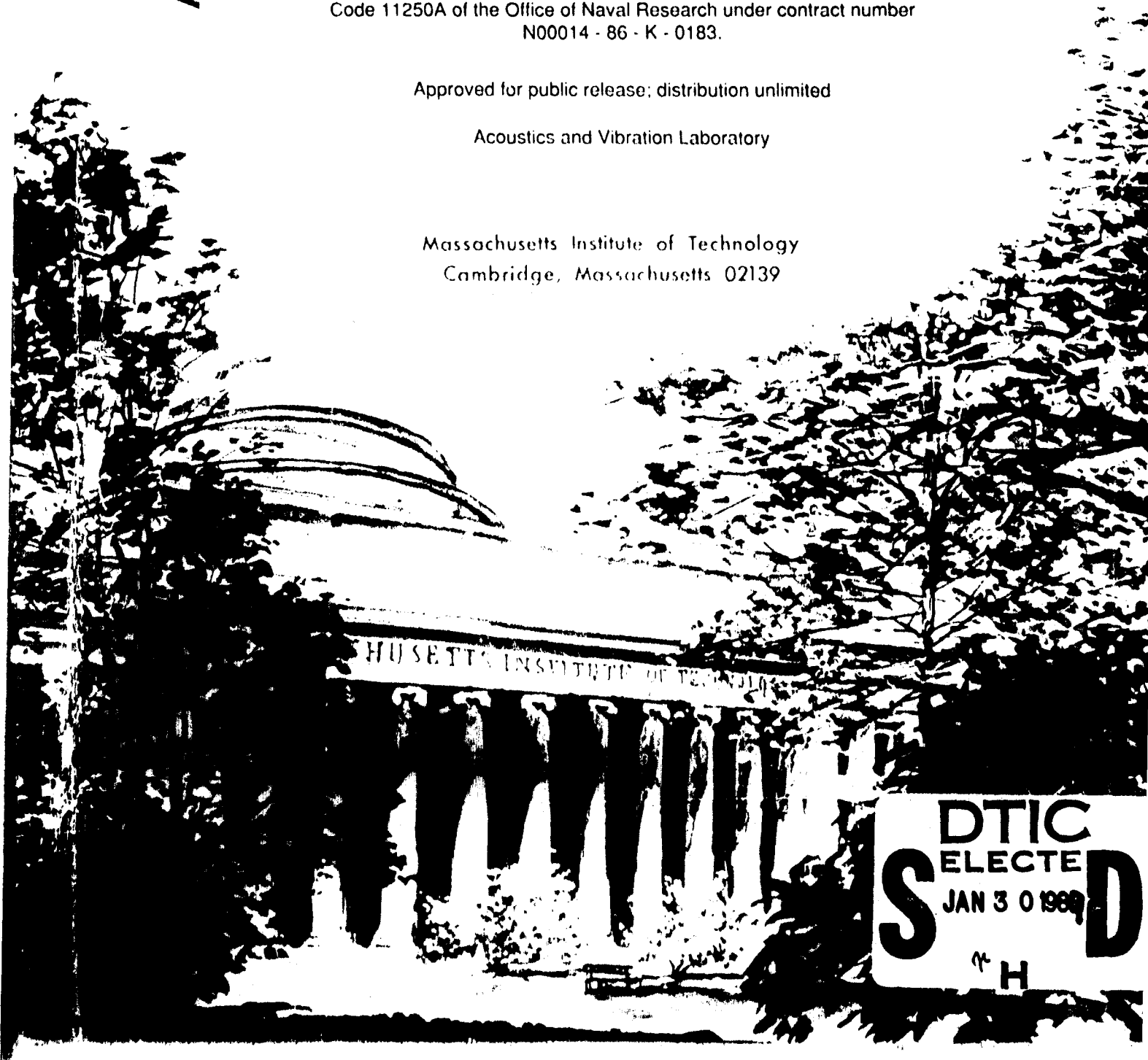
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The state of the art in measurement and interpretation of dynamic wall pressure beneath a turbulent boundary layer is reviewed. The mean pressure increase for shear flow over an orifice in a wall is explained, using triple deck theory, to stem from streamline contraction resulting from removal of the no-slip boundary condition. The effect in viscous hole diameters is too small to suggest that the dynamic pressure increase reported by Bull and Thomas (1976) for flow over a "pinhole" microphone stems from this mechanism. It appears that any failure (continued on reverse side)		

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ABSTRACT (continued)

→ of high frequency spectra to collapse when made non-dimensional on inner wall variables is more likely due to transducer proudness or to error in the measurement of mean wall shear stress.

The Corcos model is shown to be inadequate to describe cross-spectrum measurements. Both amplitude and phase depend also on the ratio of transducer separation to displacement boundary layer thickness. ~~DE 5~~ ←

The direct measurement of wavenumber-frequency pressure spectra in wind tunnels and on buoyant bodies is shown to scale best on $M^2 \tau_w^2$ where M is the flow Mach number and τ_w is the mean wall shear stress. These results refute past criticisms that wind tunnel measurements were facility noise dominated. Moreover, they suggest strongly that the low wave number spectra result primarily from grazing radiation from fluctuating wall shear stress dipoles in accordance with the theoretical prediction of Landahl (1975).



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DYNAMIC WALL PRESSURE MEASUREMENTS

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INTRODUCTION

The quest for information on the dynamics of wall pressure fluctuations has been motivated by two principle objectives. The first is to gain more fundamental understanding of the turbulent boundary layer, in particular, the mechanism by which it continually regenerates itself. The wall pressure measurement has the advantage of being, for the most part, non-invasive. On the other hand, it is a weighted integral of the velocity fluctuations in the boundary layer, hence, its picture of boundary layer activity must of necessity be somewhat diffuse. Pressure measurements have been combined with other measurements, in particular, fluctuating velocity and fluctuating wall shear stress. These have taken the form of long-time averages, such as in cross-correlation and cross-spectral density measurements. Short time conditional average measurements have also been taken wherein one or the other of the physical quantities has served as the trigger. A blend of the two techniques has been employed in measuring wall pressure fluctuations in the transition zone between laminar and turbulent flow. The pressure measurement is today an integral part of any serious experimental study of boundary layer dynamics.

The second objective in studying wall pressure fluctuations is structural excitation. To this end we have seen investigators turn away from cross-correlation and cross-spectral density measurements to the direct measurement of wavenumber frequency pressure spectra as being the quantity of predominant interest, particularly for underwater applications.

There have been three recent surveys of wall pressure dynamics. Two with particular emphasis on measurements are those of Willmarth (1975) and Blake (1983). The recent two-volume compendium on flow

noise by Blake (1986) contains an extensive discussion of both theoretical and experimental results on wall pressure in the second volume. It would be pointless to duplicate these efforts here; they are too recent to warrant another general survey at this time. We therefore shall concentrate on a number of specific questions related to the resolution of high wavenumber components of spectra, the interpretation of cross-spectral density measurements, and the scaling of low wavenumber components of spectra. Our emphasis shall be on outstanding problems of experiment and theory and their relationship to one another. We shall refer only to those papers that specifically deal with the questions at hand and make apologies at this time to the many substantial contributions that are somewhat apart from the central themes of this paper. We also shall resist the temptation to expand our study to a number of interesting related phenomena, such as wall pressure fluctuations in the transition zone, acoustic radiation from the transition zone, wall pressure fluctuations behind backward-facing steps, and the effects of mean pressure gradients and surface roughness on wall pressure. These matters are of substantial practical interest in their own right, but to deal with them would lead us too far astray from our major objective.

MEASUREMENTS AT HIGH WAVENUMBERS

The high wavenumber portion of the wavenumber-frequency wall pressure spectrum $\phi_p(k_1, k_3, \omega)$ is of primary interest in the determination of the mechanism of turbulence generation. The low wavenumber portion on the other hand, is more responsible for structural excitation. It has not been customary to attempt to measure this high wavenumber portion directly. Since the high wavenumber components are believed to be generated by eddies convecting in the equilibrium layer, it has been customary to predict their behavior either directly from the frequency spectrum using a measured convection velocity and Taylor's hypothesis or by Fourier transforming in space the measured cross-spectral density $\phi_p(r_1, r_3, \omega)$. Generally, only the streamwise quantities k_1 and r_1 are so treated.

It has long been known that the finite size of a pressure transducer limits our ability to resolve the high wavenumber portion of the spectrum. In essence, a flush-mounted sensor cannot resolve pressure scales that are smaller than its effective diameter. A number of corrections have been devised to account for this lack of

resolution, e.g., Corcos (1963) or Willmarth and Roos (1965). These resolution analyses have been based upon assumed forms of the wavenumber-frequency spectrum, especially for wavenumbers near the convective ridge, $k_1 = \omega/U_c$. Since these assumptions are based upon measurements of cross-spectra which themselves have been measured by pairs of transducers of limited resolving capability, the process is akin to lifting oneself by one's own bootstraps. Because of this difficulty experimentalists have continued to develop and utilize transducers with smaller and smaller effective sensing diameters. To a certain extent, this effort has been somewhat self-defeating. For as sensitivity decreases with size, it is necessary to test at higher and higher freestream velocities, i.e. Reynolds numbers, which resulted in a further decrease in the scale size of eddies. Generally these transducers were specially developed, one-of-a-kind type, either piezoelectric, Bull and Thomas (1976), or of the solid dielectric (Sell) type, Schewe (1983). Other investigators, notably, Blake (1970), Emmerling (1973), Burton (1974) and Farabee and Geib (1975), endeavored to maintain sensitivity while at the same time reducing the effective transducer diameter. These latter investigators used the same type of transducer, a Bruel & Kjaer 1/8" condenser microphone. This transducer has excellent pressure sensitivity and very low acceleration sensitivity, qualities prized for wind tunnel testing. For wall pressure measurements the conventional slotted cap on this transducer is replaced by a solid cap with a single pinhole drilled in the center. This pinhole has a diameter of 1/32". There is also a small cavity between this cap and the metal diaphragm of the condenser microphone. As a result, a Helmholtz resonator is set up between the pinhole and the cavity. The measured frequency of this resonance is approximately 17kHz, Blake (1970). Generally this frequency is much too high for the Helmholtz resonator to have any direct effect upon measured wall pressure spectra. There is one weakness to the arrangement, however. During humid weather moisture tends to collect under the cap, causing intermittent breakdown in the air dielectric with resultant intermittent high frequency bursts of noise.

The use of the pinhole microphones revealed for the first time the very substantial high frequency content of the wall pressure spectrum, see Willmarth (1975) for a detailed discussion. These results, however, were brought into question by Bull and Thomas (1976). Briefly, they measured the wall pressure spectrum using four transducer configurations of the same effective diameter: 1/32". The transducers were

- a) a Bruel & Kjaer 1/8" microphone with 1/32" pinhole cap;
- b) a specially designed flush-mounted piezoelectric transducer without a cap;
- c) the piezoelectric transducer with a 1/32" cap; and
- d) the piezoelectric transducer with a cap with the the pinhole filled with a silicon grease.

They found that cases (a) and (c) gave the same frequency spectra but that cases (b) and (d) gave frequency spectra that were suppressed in the high frequency ranges by as much as 5dB. They noted further that the presence of a cavity under a pinhole cap had no effect upon response. From this they concluded that the use of a pinhole cap caused an interaction with the turbulent boundary layer that was not related to a Helmholtz resonance but was presumably related to the local removal of the no-slip boundary condition.

One cannot take issue with the actual experiments performed by Bull and Thomas (1976) as more fully described by Thomas (1977). They appear to have been done with a meticulous attention to calibration, both in a shock tube and in a comparative acoustic calibrator. Two troublesome matters, however, remain. The first is

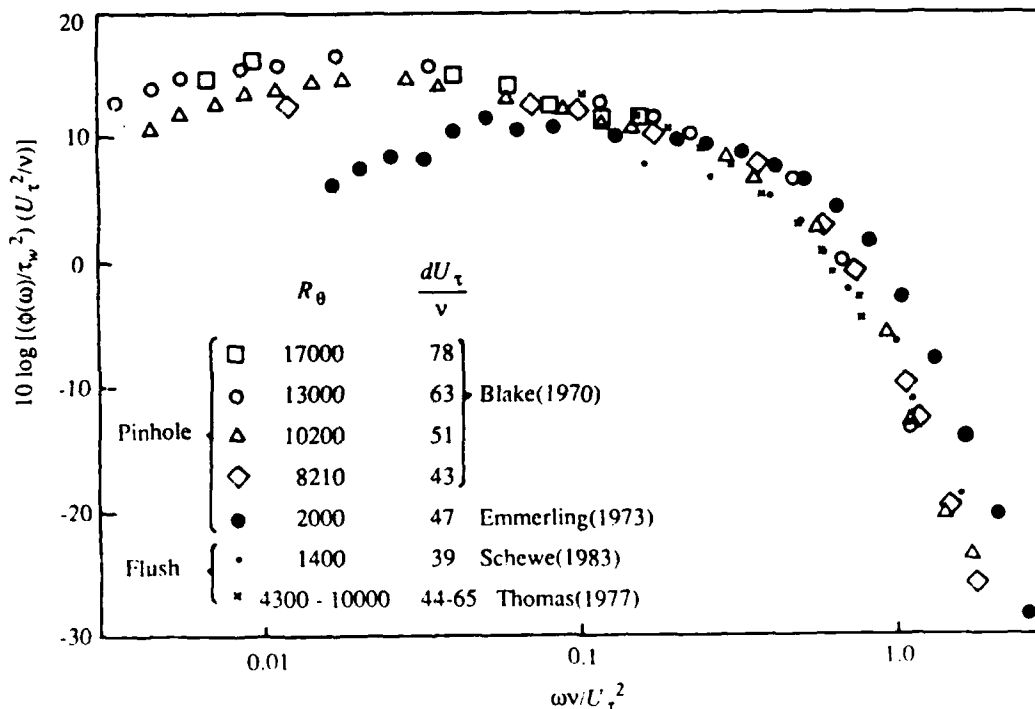


Figure 1. Wall pressure spectra measured with two types of transducers.

that a comparison of spectra measured with pinhole capped and flush mounted transducers does not show a clear distinction between the results for the two classes of transducers. Figure 1 shows a comparison of measurements of four investigators, two using the same type of B & K 1/8" microphone with a 1/32" pinhole cap and two using flush mounted transducers. All measurements were made in essentially zero pressure gradient turbulent boundary layers over a range of Reynolds number $U\Theta/\nu$ where Θ is the momentum thickness. The spectra are non-dimensionalized using wall variables u_τ and ν . Such variables are generally considered preferable for high wavenumber components of the spectrum generated in the equilibrium portion of the boundary layer. Blake's and Emmerling's measurements were done with the pinhole microphone. Schewe's measurements, were carried out with a specially designed Sell microphone. Bull and Thomas used a piezoelectric transducer. The effective diameters, d^+ , covered nearly the same range. It is interesting to note that Blake's spectra are actually somewhat lower than that of Schewe at high frequency in spite of the fact that Blake used a pinhole transducer and Schewe used a flush transducer. Clearly factors other than the pinhole are involved.

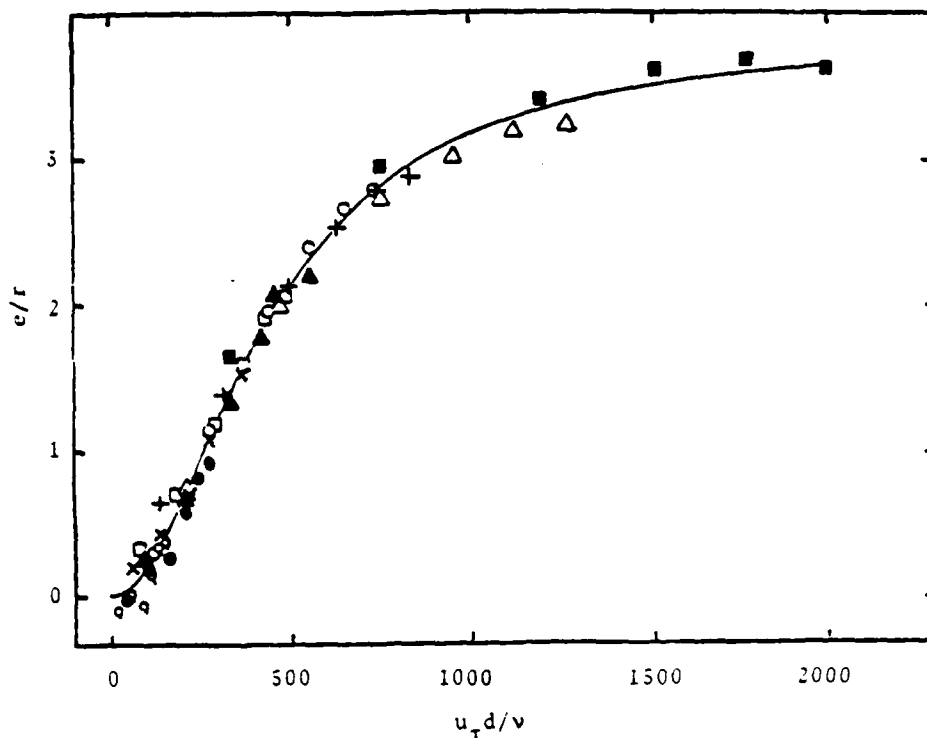


Figure 2. Apparent pressure increase e as a function of orifice diameter d , from Franklin and Wallace (1970).

The second matter is the difficulty in establishing a physical basis for the interaction caused by the pinhole microphone with the turbulent boundary layer. It is rather well known that an orifice in a wall beneath a turbulent boundary layer measures a mean pressure higher than the true mean pressure at the wall. Figure 2 taken from Franklin and Wallace (1970) shows the apparent pressure increase e divided by the mean wall shear stress τ_w as a function of the orifice diameter d^+ in viscous units. Franklin and Wallace do not give a physical explanation for this effect. It can be demonstrated qualitatively by utilizing the results of triple deck analysis of laminar flow over a trailing edge. Messiter (1971) shows that the pressure distribution in the outer (potential) deck of the flow downstream of a trailing edge is given by the expression

$$\frac{\bar{p}}{q} = \frac{A_1 R_L^{-1/2}}{3\sqrt{3}} x^{-2/3}, \quad A_1 = 1.2881$$

where the Reynolds number, R_L is based upon the plate length L and q is the dynamic head. This positive pressure is impressed across the decks to the wake centerline.

Upstream of the trailing edge, but not immediately adjacent to it, the wall shear stress τ_w is that of Blasius flow:

$$\frac{\tau_w}{q} = \frac{0.664}{\sqrt{R_L}}$$

From these equations, we find that the mean pressure averaged over a distance d downstream of the trailing edge can be written in the form

$$\frac{\bar{p}}{\tau_w} = \text{const.} \frac{1}{d} \int_0^d x^{-2/3} dx$$

and is independent of the Reynolds number R_L . The fact that the integral diverges is of no consequence; it is well known the triple deck solution fails in the immediate vicinity of the trailing edge where it must be matched to a full solution of the Navier-Stokes equations in a domain of radius $R_L^{-3/4}$. Although the pressure singularity at the trailing edge is removed by the analysis of Hakkinen and O'Neil (1967), the task of obtaining a full solution for a three-dimensional orifice remains formidable. Nevertheless, the major results are clearly correct; that is, there is a pressure increase behind the trailing edge and that it scales on the upstream wall shear stress. Since this pressure increase is associated with

the convergence of the mean flow streamlines behind the trailing edge, it might be conjectured that in the case of the turbulent boundary layer, this process could carry the active equilibrium portion of the inner boundary layer in closer to the transducer, thus producing greater apparent components of the high frequency spectrum. The difficulty, however, is evident in Figure 2, wherein it is clear that the effect on mean pressure is negligible for the effective transducer diameters under consideration ($19 < d^+ < 45$) and hence would likely be unimportant for fluctuating pressures as well.

It would seem reasonable that the behavior of pin-holed microphone might be explained by experiments wherein the impedance of an orifice subject to grazing flow was measured, Ronneberger (1972), Kompenhans (1976) and Kompenhans and Ronneberger (1980). In these experiments the orifice impedance was measured by two different techniques. One involved the measurement of the pressure drop across the orifice and the velocity in the orifice. In order to measure the orifice velocity it was necessary to supply a steady flow through the orifice in order to utilize the hot wire anemometer. This was avoided in the second technique where an impedance tube was used on the side of the orifice away from the grazing flow. The results were reported in terms of the difference in orifice impedance from the grazing flow to the no-flow case. In all cases the total orifice resistance was positive, although grazing flow did reduce resistance slightly at high reduced frequencies. Since the studies were done for turbulent as well as for laminar boundary layers, it is difficult to reconcile these results with the findings of Bull and Thomas. A parenthetical remark is in order here, not because it sheds light on the performance of the pinhole microphone, but because it illustrates the subtleties in the experiments which must be considered. Kompenhans and Ronneberger measured the dc orifice resistance as a function of grazing flow velocity and, in fact, found that it agreed remarkably well over a wide reduced frequency range with the impedance tube measurements, indicating a simple quasi-steady law for acoustical resistance of the form

$$R \sim 0.17\rho_0 U_\infty$$

would be valid for reduced frequencies $\omega a/U_\infty < 0.1$ and for ratios of displacement boundary layer thickness δ^* to orifice radius a greater than 0.4. Oddly enough they did not report the pressure drop across the orifice as a function of grazing flow velocity with no mean flow

through the orifice. They simply inferred this as being zero at the limit of zero flow through the orifice, see Figure 3. Yet, for their test conditions where d^+ varied from about 160 to 2000, the pressure increase found by Franklin and Wallace would have completely swamped the pressure differences reported. The only possible conclusion that one can reach is that their static pressure tap by chance or design had the same diameter in viscous lengths as the orifice under study.

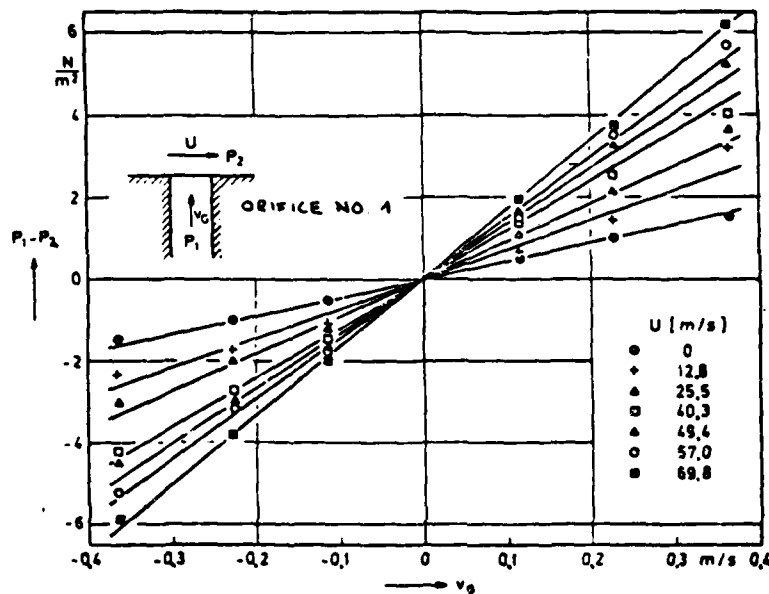


Figure 3. Pressure difference between both sides of the orifice ($d^+ = 1.9\text{mm}$). Steady state flow resistance as a function of the flow velocity, from Kompenhans and Ronneberger (1980).

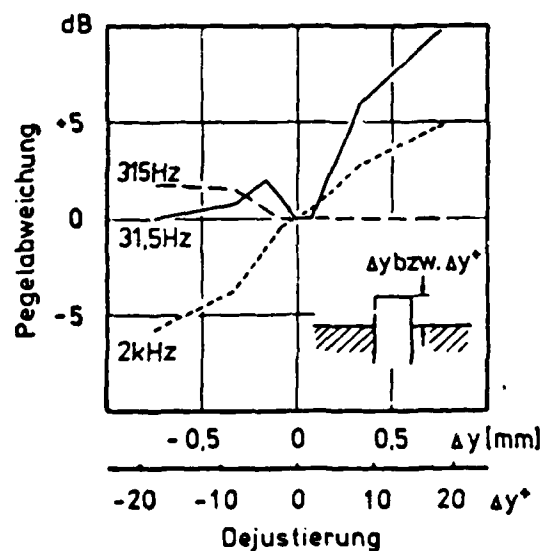


Figure 4. Effect of transducer proudness on one-third octave wall pressure levels, $d^+ = 52$, from Langeheineken and Dinkelacker (1977).

We turn our attention now to other effects which may influence the accuracy of measurement of the high wavenumber components of wall pressure spectra. The first is a question of "proudness" of any flush mounted transducer. Figure 4 is taken from Langeheineken and Dinkelacker (1978). The 1/8" Bruel & Kjaer microphone without any cap was mounted in the wall beneath a turbulent boundary layer and the protrusion or proudness of the microphone varied through positive to negative values. The influence of these variations upon the wall pressure spectrum is shown in three 1/3 octave bands as a function of proudness measured in both millimeters and in viscous lengths. Clearly a flush mounted transducer must be extremely fair in the wall, especially in high velocity turbulent boundary layers.

Since the high wavenumber portion of the wall pressure spectrum scales best on inner viscous parameters, it is important that one obtains a very accurate measure of the mean wall shear stress in the neighborhood of the pressure measurement point. Most shear stress measuring devices are non-linear. These include the surface fence, the Preston tube, the Stanton tube and the hot film anemometer. Since the dynamic wall shear stress under a turbulent boundary layer is approximately 35% of the mean (with an error of $\pm 20\%$ depending on the measurement technique) it is clear that any non-linear device will bias the mean severely. One must therefore calibrate in a turbulent flow environment such as in fully developed pipe flow and hope that the test environment has similar dynamics. Measurement of the mean wall shear stress by determining the terms of the momentum integral seem to be recommended primarily by those who have never attempted to do this. Finally, there is the simple technique and still one of the best, of fitting a semi-log plot of the mean velocity profile to the Karman slope. This works reasonably well if you have an excruciatingly accurate measure of the distance of your hot wire from the wall. Even then, one must bear in mind that the Karman slope itself is known to at best two significant figures. The scatter in the data in Figure 1 could easily be attributed to a 5% error in the measurement of mean wall shear stress.

Finally, there is a question as to whether or not the above scaling of wall pressure spectra on inner variables remains valid at very high Reynolds numbers. Efimtsov (1984) has assessed a very large body of Soviet measurements of wall pressure spectra. These cover a Mach number range from 0.015 to 4.0. He finds no Mach number effect that is independent of its influence upon wall quantities which go to make up the Reynolds number $Re = u_\tau \delta / \nu$. As illustrated in Figure 5 he finds that for $Re \gg 3 \times 10^3$, the spectra

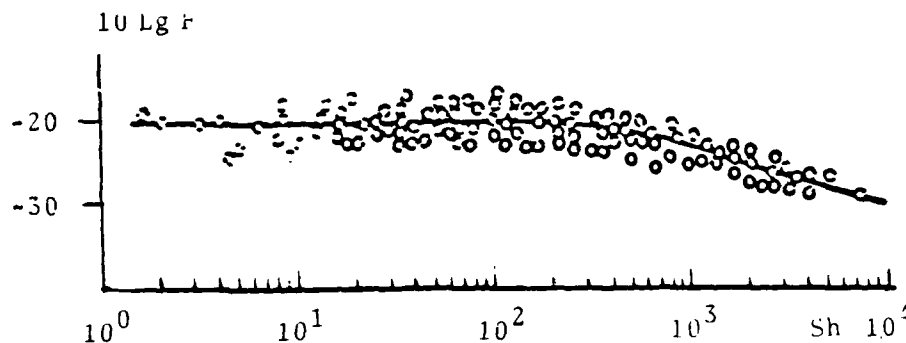


Figure 5. Wall pressure spectral density at high Reynolds number $Re = u_\tau \delta / \nu$; $F = \phi(\omega) / u_\tau^3 \rho \delta$, $Sh = \omega \delta / u_\tau$, from Efimtsov (1984).

collapse on mixed parameters $\Phi_p(\omega) / u_\tau^3 \rho \delta$ and $\omega \delta / u_\tau$. These spectra have been corrected for the effect of transducer size, unfortunately, by methods reported in publications presently unavailable to the author. For lower Reynolds numbers the high frequency spectra collapse on the usual inner parameters. The low frequency portions of the spectra do not, these being presumably influenced by the large eddies in the outer flow.

INTERPRETATION OF CROSS-SPECTRAL DENSITY MEASUREMENTS OF WALL PRESSURE

Longitudinal and lateral cross-spectral density measurements of wall pressure fluctuations serve two useful purposes. First, they are essential to any procedure for determining the resolution of a transducer of finite size. Second, they are useful for determining structural response to turbulent boundary layer excitation when the principal mechanism governing the response is that of hydrodynamic coincidence (see the next section for a more complete discussion). In this respect they are markedly superior to the older space-time correlation measurements. With questions of structural response and reradiation, it is what is going on in a given frequency band that is of practical interest. All too often correlation measurements are contaminated by extraneous frequency components.

There are two aspects of the cross-spectral density measurements which to this day are still only partially understood. The first has to do with the determination of streamwise convection velocity. This is determined by setting the phase of the longitudinal cross-spectrum equal to $\omega r_1 / U_c$ where r_1 is the longitudinal separation of the transducers. Hopefully, the

convection velocity U_c is principally a function of frequency ω only and not of r_1 , for otherwise the process would be a pointless exercise. Figure 6 shows the result of such measurements by Bull (1963) and Blake (1969). Bull's measurements were done in 1/3 octave frequency bands with transducers of an effective diameter $d^+ \sim 170$. Blake's measurements on the other hand, were carried out using a 5Hz bandwidth filter up to a frequency of 250Hz and a 50Hz bandwidth filter for higher frequencies. His transducer had a much smaller effective diameter, $d^+ \sim 45$. Although there is a good correspondence between the results of the two sets of experiments for $\omega \delta^+ / U_\infty > 1.5$, below this frequency the results diverge dramatically. Blake (1970) has argued that the reason for this discrepancy lies in the inherent dispersiveness of the wall pressure spectrum and its influence upon the differences in measurement technique. In essence, Bull measured a group velocity $U_{cg} = \Delta\omega / \Delta k$ where $\Delta\omega$ is the 1/3 octave bandwidth and Δk is proportional to the reciprocal of d^+ . Blake, on the other hand, measured something much nearer to a phase velocity $U_{cp} = \omega / k_1$. Further, it is evident that the phase velocity is increasing with frequency for $\omega \delta^+ / U_\infty < 0.5$ and hence the group velocity must be greater than the phase velocity in this range. One should also note that Blake's phase velocities increase with streamwise transducer separation r_1 / δ^+ at low frequencies.

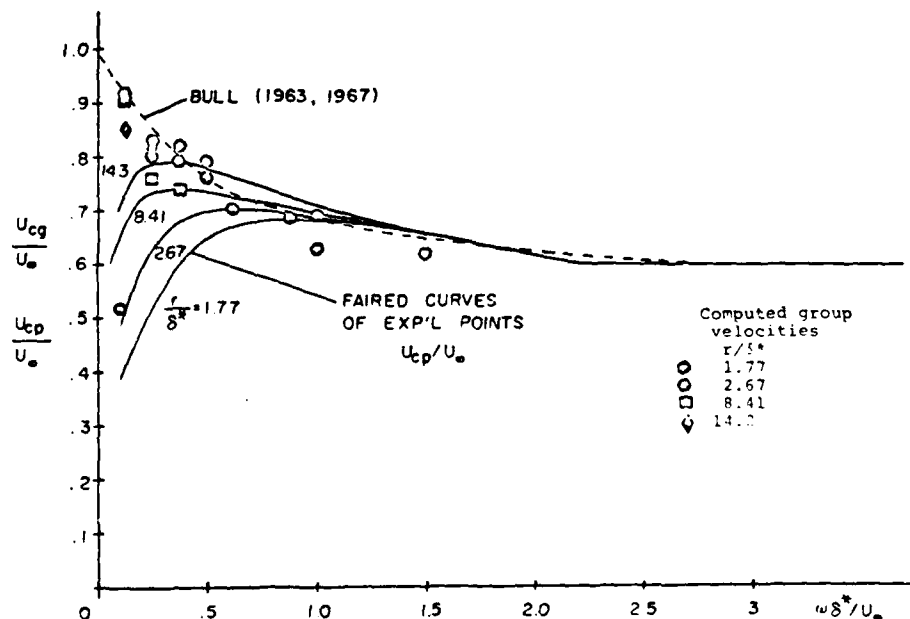


Figure 6. Comparison of phase and group convection velocities, smooth wall, from Blake (1969).

Another question has to do with the measurement of the normalized amplitude functions for the longitudinal and the lateral cross spectral density, see Figures 7 and 8 for the results of Bull (1963). These have been plotted in terms of the so-called Corcos

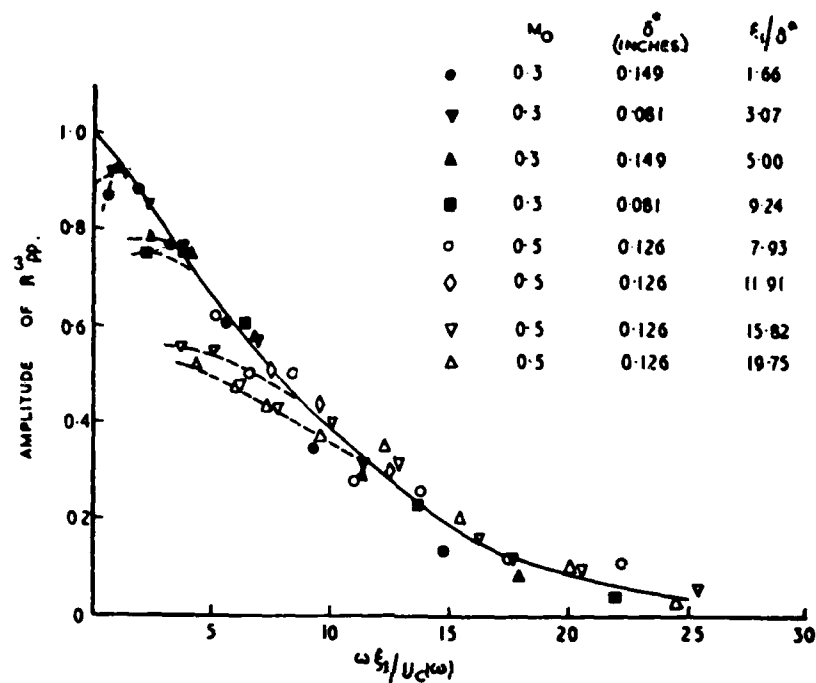


Figure 7. Amplitude of narrow-band longitudinal space-time correlations of the wall-pressure field, from Bull (1963).

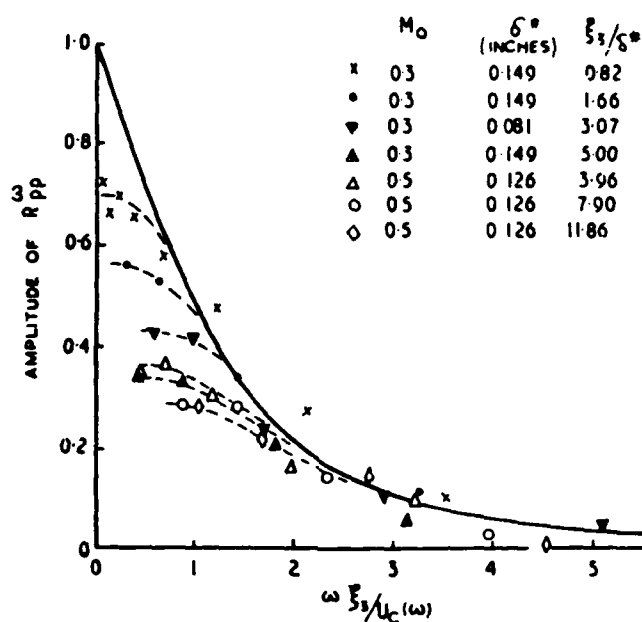


Figure 8. Amplitude of narrow-band lateral space-time correlation of the wall-pressure field, from Bull (1963).

(1963, 1967) similarity parameters $\omega \xi_1 / U_c$ and $\omega \xi_3 / U_c$. In both cases at low frequencies the amplitude functions decrease with increasing transducer separations. Blake's data on the other hand, showed no such dependencies on separation distance. Of course, one would expect the amplitude function curves to shift left at low frequencies if the group convection velocity rather than the smaller phase convection velocity is used to make the frequency non-dimensional. However, the phase velocity is closer to the group velocity for large separation distance r_1 / δ^* , an effect opposite to that which would be required to explain Bull's results in Figures 7 and 8. Recently, Moller (1987) made additional cross-spectral density measurements using methods similar to those of Blake but for significantly greater separations. He found a tendency for the the amplitude functions to fall off at low frequencies in a fashion similar to that seen by Bull. It is clear then that the Corcos similarity representation

$$\phi_p(r_1, r_3, \omega) = \phi_0(\omega) A\left(\frac{\omega r_1}{U_c}\right) B\left(\frac{\omega r_3}{U_c}\right) \exp(i\omega r_1 / U_c)$$

is inadequate. The A and B functions must also be dependent on r_1 / δ^* and r_3 / δ^* , respectively. Clearly, as r_1 approaches zero, the A amplitude function must approach one. However, this is not necessarily so if the frequency ω alone approaches zero for finite r_1 . This difference in limiting processes is masked when only the similarity parameter $\omega r_1 / U_c$ is considered. A corresponding remark applies to the lateral amplitude function. This dependency on spatial separation is probably not of too great consequence when one uses Corcos' form in determining the resolution of small transducers, Corcos (1963), but it clearly would not be appropriate to Fourier transform the same form in space in order to obtain a wavenumber-frequency spectrum. Attempts to do this have generally yielded wavenumber-frequency spectra that are too high even fairly close to the convective ridge.

LOW WAVENUMBER COMPONENTS OF TURBULENT BOUNDARY LAYER WALL PRESSURE FLUCTUATIONS

The principle interest in the low wavenumber components of turbulent boundary layer wall pressure fluctuations lies in their role as a source of structural excitation and subsequent reradiation of acoustic noise. This subject merits a discussion of its own, see

Leehey (1988). Here we shall simply state the main considerations. Assuming the frequencies in the audio range are of primary interest, at high flow Mach numbers it is possible to have coincidence between the convection speed of the wall pressure and the bending wave speeds of the resonant structural modes. When this occurs there is very substantial power flow from the boundary layer to the structure. This mechanism, termed hydrodynamic coincidence, is of considerable importance when dealing with aircraft cabin noise or with vibration of internal electronic components of rockets. In underwater applications, the frequencies of interest are approximately the same but the Mach number of the mean flow is two orders of magnitude less. At the same time the scantlings of a sonar dome are not terribly different from those of an aircraft hull structure. Consequently, the hydrodynamic coincidence mechanism is (ironically) unimportant in underwater applications. Since substantial sonar self-noise appears to be attributable to the turbulent boundary layer over the sonar system, we must look for another mechanism. Attention has therefore by default been directed towards the low wavenumber components of the wall pressure spectrum. They seem to be "the only game in town." As we shall see, they are exceedingly weak, approximately 35dB below the level of the convective ridge, $k_1 = \omega/U_c$, but they are capable of driving effectively both resonant and non-resonant structural modes with long bending wavelengths. In a sense, this is the converse of the resolution question discussed earlier.

The low wavenumber components have received substantial theoretical attention over the past quarter of a century. Phillips (1955) and Kraichnan (1956) considered the limit process where first the Mach number approached zero and then the wavenumber magnitude approached zero. For this limit process the wavenumber magnitude of the wall pressure spectrum approached zero as the square of the wavenumber. An equivalent statement is that the integral correlation area must vanish. Should a Taylor hypothesis based on freestream velocity be appropriate to this circumstance, the result would also say that the low frequency spectrum should vanish as the square of the frequency. Considerable experimental effort was expended early on to verify the last prediction. Indeed, in one or two cases it appeared that the frequency spectrum did tend to turn down as frequency was reduced to very low frequency. The alternative limiting process is to let the wavenumber approach zero and then let the Mach number approach zero. Compressibility remains important in this process for one passes the acoustic wavenumber

$k_{3c} = \omega/c$ on the way to zero wavenumber. In general, the limit processes are not interchangeable, see Chang and Leehey (1979) for an illustration in the closely related physical problem of predicting the radiation impedance of a resonately vibrating plate subjected to low Mach number grazing flow. Furthermore, it is the second limiting process which is physically meaningful in application. It becomes necessary to consider acoustic radiation from the boundary layer.

Powell (1960) made a prediction of the acoustic radiation to the far field away from a turbulent boundary layer over a rigid infinite plane wall. By neglecting oscillatory shear stress gradients at the wall he was able to construct a mirror image flow on the far side of the wall which permitted him to remove the wall entirely leaving two adjacent blobs of turbulence--implicitly bounded in extent. The Lighthill (1952) theory could then be applied. It states that the intensity of farfield radiation follows a M^8 law. Clearly this result does not apply to points on the image plane between the blobs, for the intensity there is not proportional to the mean square pressure, nearfield components undoubtedly dominate.

For predicting the pressure fluctuations at the wall the same formalism has been used. An important difference, however crept in. Because of the slow downstream growth of the turbulent boundary layer it has been customary to consider it as approximately homogeneous in planes parallel to the wall. Therefore to predict the pressure at a point on the wall one is led to consider excitation from a layer of turbulent fluid of finite thickness but of infinite extent over the wall. This leads one immediately to the well-known Olber paradox: Since the amplitude at the reception point is proportional to the reciprocal of the distance the source is away but the perimeter of sources increases directly as the distance, the integrated effect over the plane is to produce an unbounded pressure at the reception point, provided the sources are incoherent. It is not possible to determine quantitative levels of low wavenumber wall pressure components for wavenumbers at and below the acoustic wavenumber under these circumstances.

The last proviso is necessary, for it is quite possible to obtain bounded pressure at the reception point where there is infinite plane of coherent sources. For instance, consider the Rayleigh formula:

$$p(\vec{x}, t) = \frac{\rho_0}{2\pi} \int_S \frac{v_t(\vec{y}, t-r/c)}{r} d\vec{y},$$

$r = |\vec{x} - \vec{y}|$, for the pressure p at point \vec{x} from a normal velocity field v over a portion S of an otherwise infinite plane rigid baffle. Move \vec{x} to the plane and let S be a rigid piston of infinite extent oscillating at velocity $v_0 e^{-i\omega t}$. Writing $p(\vec{x}, t) = p_0 e^{-i\omega t}$, we have

$$p_0 = \frac{-i\omega\rho_0 v_0}{2\pi} \int_S \frac{e^{ikr}}{r} d\vec{y}, \quad k = \omega/c,$$

$$= -i\omega\rho_0 v_0 \int_0^\infty e^{ikr} dr$$

in polar coordinates. But in terms of generalized functions

$$\int_0^\infty e^{ikr} dr = ik^{-1} + \pi\delta(k),$$

Jones (1966, p. 469), hence

$$p_0 = \rho_0 c v_0$$

the expected plane wave result. Thus when distant sources, $v_0 e^{-i\omega t}$, are completely coherent, the pressure is bounded at the measurement point.

Bergeron (1973) showed that the wavenumber spectrum could be bounded at the acoustic wavenumber by considering a finite domain of turbulence. Ffowcs Williams (1982) extended his own (1965) results and those of Bergeron to obtain the following representations of the effect of compressibility for various ranges of low wavenumbers. The wavenumber-frequency spectrum can be written for

$$k_\alpha^2 \ll \omega^2/c^2 \ll \Delta^{-2}$$

as

$$P^*(k_\alpha, \omega) = \rho_0^2 U^3 \Delta^3 (U/c)^2 (\omega\Delta/U)^2 F_1(\Delta k_\alpha, \omega\Delta/U)$$

where Δ is the boundary layer thickness, U the free stream velocity and k_α is a two-dimensional vector in the plane of the wall. The

non-dimensional spectrum F_1 is devoid of compressibility effects and is presumed non-vanishing as $k_\alpha^2 \rightarrow 0$. Then P^* is proportional to the square of the Mach number $M = U/c$ in this limit.

$$\text{For } \omega^2/c^2 \ll k_\alpha^2 \ll \Delta^{-2},$$

$$P^*(k_\alpha, \omega) \sim \rho_0^2 U^3 \Delta^3 (\Delta k_\alpha)^2 F_2(\Delta k_\alpha, \frac{\omega \Delta}{U}).$$

This is the incompressible limit of Phillips and Kraichnan. Finally, in the neighborhood of the acoustic wavenumber ω/c ,

$$k_\alpha^2 \sim \omega^2/c^2 \ll \Delta^{-2},$$

$$P^*(k_\alpha, \omega) \sim \rho_0^2 U^3 \Delta^3 \ln\left(\frac{R}{\Delta}\right) \left(\frac{\omega \Delta}{c}\right)^4 \delta\left((\Delta k_\alpha)^2 - \left(\frac{\omega \Delta}{c}\right)^2\right) F_3\left(\Delta k_\alpha, \frac{\omega \Delta}{c}\right)$$

where the turbulence is limited to within a radius R and a near field of radius Δ has been excluded, both measured from the reception point. Again the non-dimensional spectra F_2 and F_3 and presumed to be innocuous.

The presence of the Dirac delta function in this last formulation has been termed by Ffowcs Williams as an expression of "the individual resonance structure of acoustically coincident elements." From another point of view, it seems that it has nothing to do with resonance, but is a evident result of the extraction of all nearfield components from the excitation. Thus we have the spatial "pure wave" equivalent of the temporal "pure tone," which must appear as a delta function in the wavenumber spectrum. It is perhaps worth mentioning that this form of the spectrum could never be measured by any line array or similar wavenumber filter as such filters measure the trace wavenumber rather than the magnitude $|k_\alpha|$ of the wavenumber and consequently will produce a result which spans all wavenumbers from zero wavenumber up to the acoustic wavenumber.

Howe (1987) has performed an independent analysis of the behavior of the spectrum in the neighborhood of the acoustic wavenumber. His results appear similar to those of Ffowcs Williams with the exception that the excluded nearfield region is of the order of an acoustic wavelength rather than of the order of a boundary layer thickness. Ffowcs Williams (1982) extended the above results to incorporate a Corcos similarity hypothesis in the analysis, applying this hypothesis to the turbulent sources rather than to the wall pressure field itself. This results in the

wavenumber-frequency spectrum decaying for fixed wavenumber as ω^{-2} . As we have mentioned in the previous section, the Corcos hypothesis does not seem to be appropriate for wavenumbers of the order of the reciprocal of the boundary layer thickness. It seems that future analyses should concentrate more on this domain than simply the effects of compressibility. Howe (1987) has also considered the role of surface curvature in bounding the wavenumber spectrum in the vicinity of acoustic wavenumber accounting for the creeping transmission of grazing acoustic waves over the curved surface. As we have suggested earlier, another mechanism, that of requiring coherence of sources in the far distance would also provide a bound on the spectrum.

Attention has also been given to the heretofore neglected wall shear stress fluctuations. These seem to have a chameleon behavior and the theoretical results are a mixed bag. The shear stress fluctuations on one hand can be considered as dipole radiators with their principle axes in the plane of the wall. On the other hand, they may be considered to dissipate radiation through the mechanism of the viscous mode in the immediate vicinity of the wall. Landahl (1975) predicted them to be an important source of radiation, Howe (1979) found only the dissipation mechanism to be significant, Haj-Hariri and Akylas (1985) seem to take a position somewhat in between. They find a small contribution to low wave number wall pressure from shear stress dipoles.

There is an essential difference in the analysis of Landahl from those of Howe and Haj-Hariri and Akylas. Landahl's analysis is constructed in such a way as to approximately determine the effect of sublayer bursting upon wall pressure. The other analyses do not specifically treat this mechanism. Although none of the analyses establish quantitative levels, the differences in approach affect the scaling laws for the wall pressure spectrum. Landahl estimates the intensity of acoustic radiation from the Lighthill stresses in the boundary layer as $l - \tau_w u_r M_r^5$ where M_r is the Mach number based on friction velocity. The intensity of acoustic radiation from the wall shear stress contributions is estimated as $l - \tau_w u_r M_r^3$. Neither of these estimates give any indication of directivity, nor do they account for absorption of grazing acoustic waves from these sources at the wall. Neglecting these effects, we see that the contribution from distant turbulent boundary layer bursts to the mean square wall pressure must be of the order $\tau_w^2 M^4 (u_r/U_\infty)^4$ for the Lighthill stresses, or the quadrupole sources, and of the order $\tau_w^2 M^2 (u_r/U_\infty)^2$ for the fluctuating wall shear stress, or dipole sources. Since

these are radiation effects, they must apply to wavenumbers at and below the acoustic wavenumber. Clearly the radiation from wall shear stress sources dominate that from Lighthill stresses at low Mach numbers. We shall refer to this scaling again in the section immediately following on experimental results for the low wavenumber wall pressure spectrum.

One especially useful theoretical result is by Ffowcs-Williams (1965) in which he points out that for sources within the boundary layer one can expect that if their wavenumbers are greater than the acoustic wavenumbers, they experience exponential decay towards the wall. This characteristic, well known in many acoustic radiation problems, is of considerable utility in dealing with problems of sonar self-noise. For example, it is not unrealistic to place what is known as an outer decoupling coating between a transducer system and the exciting wall pressure field. One can expect in the absence of other difficulties, such as discontinuities in either the inner or outer structures, that there will be a very substantial exponential decay of the exciting wall pressure fields. This together with the reduction of self-noise which is inherent in the inability of a transducer of finite size to resolve pressures from small scale turbulent eddies are perhaps the two most effective mechanisms by which sonar self-noise resulting from turbulent boundary layer excitation is presently controlled. Unfortunately neither technique is effective for excitation wavenumbers at and below the acoustic wavenumber.

Two principle techniques have been used for the measurement of the wavenumber-frequency spectrum of wall pressure. One is to mount a thin plate or membrane flush in the wall of the wind tunnel test section and to measure the response of this structure to turbulent boundary layer excitation. The response is measured at the resonant frequencies of the structure, either by a small accelerometer or by a non-contact displacement gauge. One also requires the precise knowledge of the modal pattern and the damping of that mode at resonance. Frequently the plate is elongated in the flow direction to the extent that only the zero lateral wavenumber component contributes to the response. As many as thirty longitudinal components have been measured. For use of this method, see Martin and Leehey (1977), Jameson (1970) and particularly Martin (1976). The other technique is to use a flush-mounted array of microphones, usually of 1" diameter, set as close together as possible, usually aligned in the streamwise direction. Generally the outputs are summed with alternating phase. The array typically consists of six

or twelve elements. Again, in the usual measurement, only the zero lateral wavenumber contributes. For applications of this technique, see Farabee and Geib (1975) or Martini, Leehey and Moeller (1984). The structural transducer is certainly considerably cheaper than the array. It is somewhat more difficult to calibrate. The array is subject to spatial aliasing, which is only partially alleviated by using the largest possible microphone elements and setting them as close together as possible. The structure does not alias. It is equivalent in the spatial domain to a continuous analog time signal. On the other hand, the spatial side lobe structure of the microphone array can be controlled by shading the array, that is, by adjusting the sensitivities of the individual array elements. In both techniques, the resulting signal is frequency analyzed.

The major problem with any wavenumber measurement technique is spatial side lobe contamination. This takes two forms, the high wavenumber side lobes are contaminated by the wall pressure signal at the convective ridge. The side lobes below the main wavenumber side lobe are contaminated by acoustic signals. A plate transducer is particularly good at the rejection of convective ridge contamination. On the other hand, its response cannot be shaded, thus precluding the investigation of acoustic contamination problems by array shading. Finally, it is possible to insert time delays between the microphone responses of the microphone array, thus permitting array steering and the determination of localized acoustic sources in the test environment. A point to bear in mind, but one which is not of great consequence at the current state of wavenumber measurements is that the techniques are incapable of distinguishing between positive and negative wavenumbers, i.e., the measured wavenumber spectra are folded about the zero-wavenumber axis. In what follows we shall concentrate upon the longitudinal or streamwise wavenumber measurement assuming that the lateral wavenumber is set equal to zero. Further, almost all wavenumber measurements have been carried out below the convective wavenumber, as this is the domain of principle interest to structural excitation.

In all measurements to date, once the wavenumber at a given frequency is moved below the convection wavenumber, the level drops quite abruptly by as much as 35dB and then flattens off to a near constant value as far as it is possible to carry out the measurement. If one views the results at a constant wavenumber but increasing frequency there is again an abrupt fall-off from the convective ridge but a continuing dropping of level as frequency is

increased further, see Figure 9. This behavior at low wavenumbers has been termed "wavenumber white." It has become customary, therefore to present data not in isocontours that are implicit in Figure 9, but collapsed in wavenumber yielding a plot of spectrum level vs. frequency. Two families of measurements presented in this way are shown in Figures 10 and 11.

The data presented in Figures 10 and 11 come from two different wind tunnels, from two different measurement techniques both arrays and plate filters, and from widely varying test environments, ranging from closed hard-walled ducts to wall jets in a semi-anechoic chamber. One set of data is from array measurements on a buoyant body rising in water. This last set is especially significant because the environment is nearly free-field, the background noise is extremely low, and the Mach number range has been extended downward by a factor of nearly four from the wind

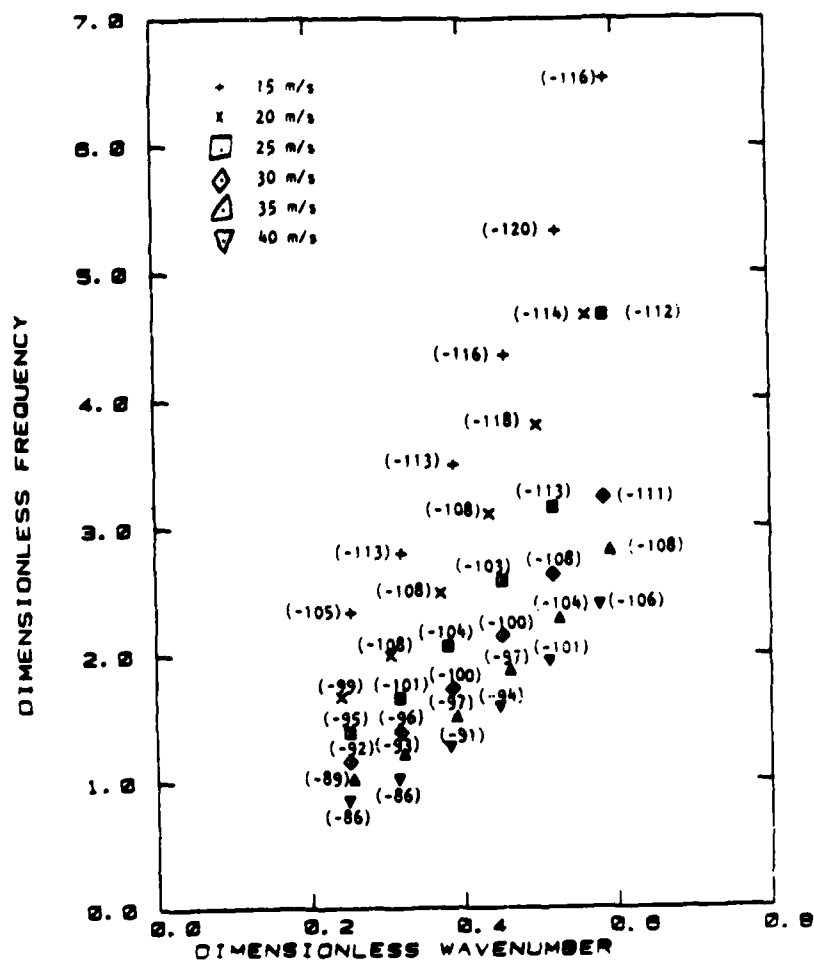


Figure 9. Low wavenumber result of Martin plate in ω - k space, from Martini, Leehey and Moeller (1984).

tunnel measurements. Considering the wide variety of test environments, it is gratifying to find that the scaling of the low wavenumber spectra on $M^2 r_w^2$ is valid.

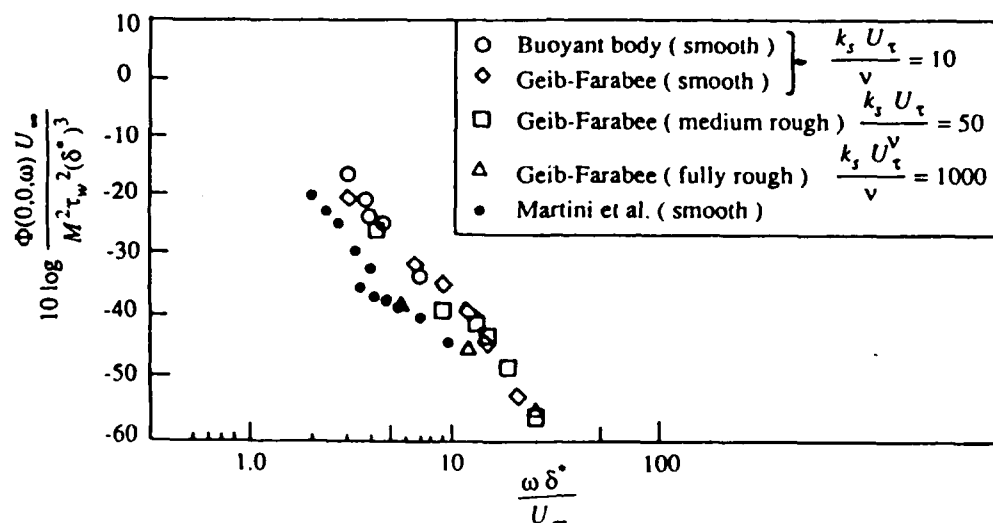


Figure 10. Wall pressure spectra at zero streamwise wavenumber. (The Mach number range extends from $M=0.03$ to $M=0.14$.)

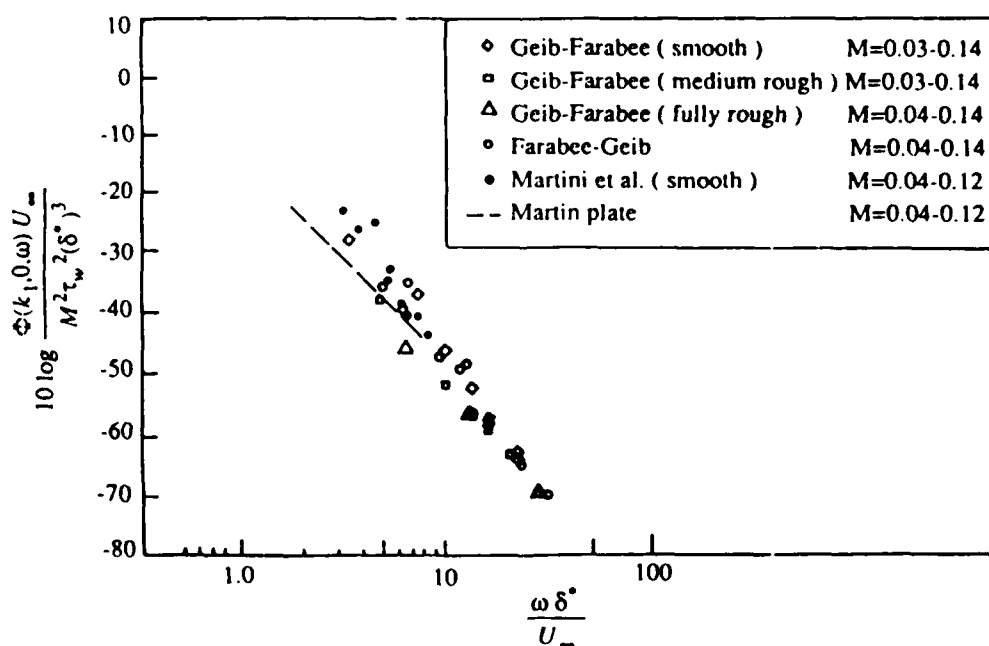


Figure 11. Wall pressure spectra for streamwise wavenumber greater than the acoustic wavenumber.

Figure 10 gives results for "summed" arrays, i.e., arrays aligned in the flow direction whose outputs are directly summed. For these arrays the outputs of the major lobes are centered on $k_1=0$ and $k_3=0$. The mean flow Mach number range extended from $M=0.03$ to $M=0.14$. It is interesting to note that the wind tunnel data of Martini et al. (1984) lie appreciably below the buoyant body data.

Figure 11 gives results for "alternating" arrays, i.e., arrays aligned in the flow direction whose transducer outputs are summed alternating in phase. Also included are results from a plate transducer, Martin (1976). Since results from a plate transducer are obtained only at resonant frequencies, a plate is similar to an "alternating" array. The major lobes for all transducers depicted in Figure 11 lay above the acoustic wavenumber.

Except for the data of Martini et al., the results for $k_1=0$ given in Figure 10 lie appreciably above those for k_1 greater than the acoustic wavenumber k_0 given in Figure 11. This rise in the wavenumber spectrum below k_0 may, however, be due to the greater susceptibility of the "summed" array to spatial aliasing.

It seems likely, today, that the low wavenumber measurements are, in fact, reflecting the behavior of the boundary layer itself and it behooves us to determine what is the true physical basis for these levels. Perhaps sufficient emphasis has already been given to the effects of compressibility and that we should be giving more attention to those wavenumbers that are commensurate with a very large scale eddy structures of the boundary layers and to the contributions of fluctuating wall shear stress. We have referred earlier to the uncertain state of affairs regarding the handling of wall shear stress. Burton (1974) found by measurements the relation $\overline{\tau_w^2} = 0.004 \overline{p^2}$ between mean square wall shear stress and mean square wall pressure. This is a difference of 24dB. Since the wavenumber white pressure levels are some 35dB below the convective ridge, it is entirely conceivable, with some allowance for viscous mode attenuation, that the low wavenumber levels are in fact set by the grazing dipole radiation from shear stress as predicted by Landahl (1975). Clearly the best experimental scaling laws for low wavenumber spectra are identical with those developed by Landahl for the influence of oscillatory wall shear stress, i.e., scaling on $M^2 \tau_w^2$. Perhaps this is an explanation as to why the scaling on M^2 extends far beyond the applicable wavenumber region suggested by the analyses of Ffowcs Williams. We note that Ffowcs Williams analyses neglected entirely the influence of fluctuating wall shear stress.

SUMMARY

We have addressed three aspects of wall pressure fluctuations created by a turbulent boundary layer. First, the resolution of high wavenumber components by very small transducers has been studied. The question of whether or not a pinhole type transducer disturbs the flow has been partially answered in the negative. Second, cross-spectral density measurements continue to show a dependence upon \bar{r}/δ^* in addition to the reduced frequency $\omega\delta^*/U_c$. These dependencies weaken the applicability of the Corcos similarity model of wall pressure especially for low wavenumber interpretations. Finally, in the matter of the direct measurement of low wavenumber components of the turbulent boundary layer wall pressure, we see that the measurement techniques and control of test environments have improved sufficiently that there is now some degree of correlation between theory and experiment. However, the essential underlying mechanism for establishing the levels of low wavenumber components remains to be determined.

We have seen that there is strong evidence to indicate the fluctuating wall shear stress may be the predominant influence in the low wavenumber wall pressure spectrum as well as in radiation from boundary layer turbulence. This places considerable importance upon quantifying experimentally the levels of wall shear stress fluctuations. In this respect the current status of research is intolerable because the ratio of rms wall shear stress to mean shear varies by a factor of approximately 10 depending on the measurement technique used.

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